Practical Problem and Solution

Problem

A work shop contains four persons available for work on the four jobs. Only one person can work on any one job. The following table shows the cost of assigning each person to each job. The objective is to assign person to jobs such that the total assignment cost is a minimum.

		Jobs			
		1	2	3	4
	A	20	25	22	28
Persons	B	15	18	23	17
	C	19	17	21	24
	D	25	23	24	24

Solution

As per the Hungarian Method

Step 1: The cost Table

		Jobs			
		1	2	3	4
	A	20	25	22	28
Persons	B	15	18	23	17
	C	19	17	21	24
	D	25	23	24	24

Step 2: Find the First Reduced Cost Table

		Jobs			
		1	2	3	4
	A	0	5	2	8
Persons	B	0	3	8	2
	C	2	0	4	7
	D	2	0	1	1

Step 3: Find the Second Reduced Cost Table

		Jobs 1	2	3	4
Persons	A	0	5	1	7
	B	0	3	7	1
	C	2	0	3	6
	D	2	0	0	0

Step 4: Determine an Assignment

By examine row A of the table in Step 3, we find that it has only one zero (cell A1) box this zero and cross out all other zeros in the boxed column. In this way we can eliminate cell B1.

Now examine row C, we find that it has one zero (cell C2) box this zero and cross out (eliminate) the zeros in the boxed column. This is how cell D2 gets eliminated.

There is one zero in the column 3. Therefore, cell D3 gets boxed and this enables us to eliminate cell D4.

Therefore, we can box (assign) or cross out (eliminate) all zeros.

The resultant table is shown below:

		Jobs 1	2	3	4
Persons	A	$\boxed{0}$	5	1	7
	B	0	3	7	1
	C	~~2~~		3	6
	D	2	0		~~0~~

$\boxed{0}$

$\boxed{0}$

Step 5:
The solution obtained in Step 4 is not optimal. Because we were able to make three assignments when four were required.

Step 6:
Cover all the zeros of the table shown in the Step 4 with three lines (since already we made three assignments).

Check row B since it has no assignment. Note that row B has a zero in column 1, therefore check column1. Then we check row A since it has a zero in column 1. Note that no other rows and columns are checked. Now we may draw three lines through unchecked rows (row C and D) and the checked column (column 1). This is shown in the table given below:

		Jobs			
		1	**2**	**3**	**4**
	A	$\boxed{0}$	5	1	7
Persons	**B**	0	3	7	1
	C	2		3	6
	D	2	0		0
			$\boxed{0}$		

Step 7:
Develop the new revised table.

Examine those elements that are not covered by a line in the table given in Step 6. Take the smallest element in this case the smallest element is 1. Subtract this smallest element from the uncovered cells and add 1 to elements (C1 and D1) that lie at the intersection of two lines. Finally, we get the new revised cost table, which is shown below:

		Jobs			
		1	**2**	**3**	**4**
Persons	**A**	0	4	0	6
	B	0	2	6	0
	C	3	0	3	6
	D	3	0	0	0

Step 8:
Now, go to Step 4 and repeat the procedure until we arrive at an optimal solution (assignment).

Step 9:
Determine an assignment

Examine each of the four rows in the table given in Step 7, we may find that it is only row C which has only one zero box this cell C2 and cross out D2.

Note that all the remaining rows and columns have two zeros. Choose a zero arbitrarily, say A1 and box this cell so that the cells A3 and B1 get eliminated.

Now row B (cell B4) and column 3 (cell D4) has one zero box these cells so that cell D4 is eliminated.

Thus, all the zeros are either boxed or eliminated. This is shown in the following table

Jobs

		1	2	3	4
	A	$\boxed{0}$	4	0	6
Persons	**B**	0	2	6	
	C	3		3	$\boxed{06}$
	D	3	0		0

Since the number of assignments equal to the number of rows (columns), the assignment shown in the above tale is optimal. $\boxed{0}$

The total cost of assignment is: 78 that is A1 + B4 + C2 + D3

20+17+17+24=78